Machian Quantum Gravity

Principal Investigator (PI): Julian Barbour

Co-Investigators: Dr Hans Westman, Perimeter Institute, Waterloo, Canada; Dr Edward Anderson, Peterhouse, University of Cambridge, Cambridge, UK and DAMTP, Centre for Mathematical Sciences, University of Cambridge, Trumpington Street, Cambridge, CB2; Sean Gryb, Perimeter Institute, Waterloo, Canada.

Project Abstract

Mach's Principle addresses the most basic issue in dynamics: the definition of motion. In 1982 Bertotti and I developed the universal method of *best matching* for constructing theories that are relational in their treatment of motion and thereby implement Mach's Principle. My recent work with Ó Murchadha and others has shown that general relativity *just* fails to be fully Machian because it is not quite scale invariant. The two main approaches to the creation of a quantum theory of gravity – string theory and loop quantum gravity – take little or no account of the specific way in which general relativity is Machian or its curious failure to be fully relational. They largely ignore its essential dynamical structure. The Machian Quantum Gravity Project aims to rectify this shortcoming by taking the relational structure of Einstein's theory as its guiding principle. It is proposed as a more fundamental *third route* to quantum gravity.

Project Summary for Lay People

Einstein's general relativity and quantum theory describe different things, gravity and atoms, and have remarkably different structures. To overcome this disharmony, theoreticians must unify the two theories in *quantum gravity*. This is the aspiration of string theory and loop quantum gravity, but the Principal Investigator believes that both these leading projects fail to take proper account of an essential issue. He has spent many years studying the foundations of general relativity, in which Einstein sought to find an alternative to the absolute space introduced by Newton to define the motion of bodies. Being invisible, this problematic concept was criticized by Mach (1883), who argued that the positions of bodies are determined *relative to each other*. Einstein attempted to implement this idea, now known as Mach's Principle, but did so indirectly and thus created confusion despite the great success of his theory. The PI and his collaborators have clarified the precise manner in which motion is relative in Einstein's theory and thereby identified its irreducible essential principle. The aim of the Machian Quantum Gravity Project is to use this insight to unify the principles of quantum theory and general relativity. It will be a *third route* to quantum gravity.

Machian Quantum Gravity: Project Narrative

Contents

1 Overview

My project scope and funding requirement are now greater than in my initial application. My collaborator Edward Anderson has already largely solved the problem on which I had intended to work, and the number of researchers seriously interested in my approach has grown significantly. Hans Westman (Perimeter) and I now collaborate; he is short listed for a distinct project, but his investigations are likely to shed light on my project. With him and others, I foresee progress on a broad front. My modified proposal addresses some of the most fundamental issues in classical and quantum dynamics from the hitherto largely ignored Machian perspective and is unlikely to obtain funding from traditional sources.

I assume the FQXi referees will be familiar with canonical quantum gravity. To keep the narrative as short as possible, I avoid too many technical details and give only application-specific references, mostly available online. However I think it is necessary to give enough detail to show that Machian Quantum Gravity (MQG) is a new and viable project and could be a better route to quantum gravity than either string theory or LQG. I argue that a major reason for the slow progress in quantum gravity has been the failure to recognize properly the manner in which general relativity is Machian, or background independent. Proponents of LQG rightly highlight the background independence but in my view fail to characterize it accurately. We are dealing with the foundations of dynamics, so this is important.

Mach's Principle has been the source of much confusion; Mach [1] was not precise. I give and argue for a unique definition and present *best matching* [2, 3] as the universal method to implement it. Best matching eliminates both absolute space and time cleanly if the universe is a closed dynamical system and, being universal, creates many Machian models, all useful in closed-universe quantum studies. Examples will be given. If the universe is not closed, the Machian perspective is still illuminating.

By my definition general relativity is Machian to a remarkable degree but *just* falls short of being perfectly so. I draw two conclusions: 1) the Machian approach can be usefully applied to general relativity; 2) one has a useful hint of how it could be modified to make it perfectly Machian and thereby eliminate its residual absolute structure.

In my narrative I shall indicate how the Machian perspective opens up new research projects and eliminates long-standing confusions. I shall conclude by listing the topics that I wish to address with my collaborators and the research methodology to be adopted.

2 Definition of a Machian Theory

In order to formulate the law of inertia, the ultimate basis of all science, Newton introduced absolute space and time, but, being invisible, they were problematic. Mach [1] argued that motion defined relative to all bodies in the universe could appear to be governed locally by Newton's framework. This qualitative idea inspired Einstein to create general relativity, but he did not develop an explicit construction that could not fail to implement it. For accidental historical reasons, he proceeded indirectly without first defining Mach's Principle, which created confusion. We need a precise definition; it requires motion, more generally change, to be defined relative to observable entities. This will be done first for particles in Euclidean space, the framework in which Mach formulated his idea, and then for the variation of fields and geometry.

2.1 Geometry and Relative Configuration Spaces

The first notion is a possible configuration of a universe, illustrated for point particles as follows. Suppose $N(N-1)/2, N \geq 5$, positive numbers satisfy the equations connecting the separations r_{ij} of N points in Euclidean 3-space. A *geometrical principle of unity* 'binds the numbers together'. Universum means a collection bound in a whole. The binding is vital.

The set $\{r_{ij}\}$ defines a configuration that can be represented by: 1) the original numbers; 2) $3N$ Cartesian coordinates, which introduce a six-fold redundancy because the choice of the coordinate origin and orientation can changed without changing the r_{ij}; 3) by a minimal set of $3N-6$ numbers from which all the r_{ij} can be constructed. The Cartesian representation is computationally by far the most convenient but comes with the redundancy.

The r_{ij} correspond to lengths measured by rods and thus to epistemological facts. However, the length ascribed to a rod is arbitrary; the r_{ij} contain an arbitrary scale. Only length ratios and angles are objectively real; the Cartesian coordinates therefore include a 7-fold degeneracy.

Many different sets $\{r_{ij}\}$ satisfy the same the principle of unity. There is space of corresponding configurations for each N. Such a configuration space, which is the basis of my project, is a very general notion; it may be called an *epistemological space* (since it contains only directly observable quantities). If defined with an absolute scale, I shall call it a *relative configuration space* $\mathcal{R}$; if without, a *shape space* $\mathcal{S}$; each point $s \in \mathcal{S}$ is a shape of a universe. All shapes are instantiations of the same principle of unity. I identify them with instants of time.

Because the different positive-number sets $\{r_{ij}\}$ satisfy the same principle of unity but define different shapes, we may call them variables. They are more fundamental than the Cartesian coordinates or some minimal set of variables because they are observational facts and express their principle of unity intrinsically. Mathematicians given Cartesian coordinates or a minimal set of variables could not interpret them without additional information, but the original numbers 'carry their own semantics'. They are *true variables.* They cannot all be independent; if they were, nothing would bind them together. That is why I call them variables and not degrees of freedom.

In this approach space is not a background in which particles reside; it is a mode of expression made possible by the unity of the true variables. It will be the basis of everything. It is in this respect that my approach differs most strongly from loop quantum gravity (LQG), in which the fundamental entities are spin networks embedded in a bare manifold. The aim in LQG is to recover the spatial interconnectedness that I presuppose from a quantum theory of gravity. I am not sure that can be done, and one of my research projects is to see how far one can get with a conservative attitude to geometry.

There is a universal formal definition of shape spaces. For N particles in Euclidean space, one first quotients the $3N$-dimensional Newtonian configuration space by the Euclidean translations and rotations, obtaining the $3N-6$-dimensional *relative configuration space* $\mathcal{R}$ (RCS) [2]. Its quotient by Euclidean dilatations is the $3N-7$-dimensional *shape space* $\mathcal{S}$ [3]. There are two reasons to stop quotienting here: 1) no dimensional quantities dependent on human conventions can appear in any $\mathcal{S}$, so that dynamics in it will employ only objective quantities – there will be no gap between epistemology and formalism; 2) fermions described by finite-component spinors occur in nature, and the minimal structure needed to represent them is an orthogonal triad (one needs orthogonality but not orthonormality). This suggests that angles belong to the bedrock of the universe.

To obtain the configuration spaces needed to study dynamical geometry, we start with Riem, the space of all suitably continuous Riemannian 3-metrics g_{ij} on a closed manifold without boundary, say S^3. Riem has six degrees of freedom per space point; its quotient with respect to the 3-diffeomorphisms of S^3, is *superspace*, the space of all possible Riemannian 3-geometries. It has three degrees of freedom per space point. To eliminate the scale in superspace, it is natural to quotient as well by general conformal transformations, which leave angles at any space point fixed but change the local volume element. This leads to *conformal superspace* (CS) [4, 5], which has two degrees of freedom per space point; they describe the 'local shape of space'. CS is obviously analogous to $\mathcal{S}$. Finally, there is a hybrid space obtained by quotienting by *volume-preserving* conformal transformations. This may be called *conformal superspace with a total volume* (CS+V) [6] and has just one single extra global degree of freedom compared with CS. Remarkably, it appears to be the space best suited to describe the dynamics of general relativity, as will be explained below.

It is also possible to define matter fields on S^3 and obtain spaces in which the geometrical configuration spaces are extended by the matter degrees of freedom.

2.2 The Definition of Mach's Principle

After this preparation, I can define Mach's Principle, or rather a classical Machian theory:

> In a *classical Machian theory* [2] specification of a direction at a point s in some shape space $\mathcal{S}$ uniquely determines a curve in $\mathcal{S}$.[1] No speed is associated with evolution along

[1] In the case of point particles, the mass ratios of the particles are assumed known.

the curve; that would require an absolute time, which Leibniz and Mach both denied. Data in the form of a direction at a point in some $\mathcal{S}$ will be called *Machian initial data.*

Being expressed exclusively in relational terms, theories of this kind are manifestly Machian. They exist and are superior to Newtonian theories, first in lacking the disturbing gap between epistemology and ontology and second in having greater predictive power.

This second superiority illustrates clearly the difference between Newtonian and Machian theories and is worth spelling out. Poincaré [7] anticipated it in an analysis of the consequences of Newton's absolute framework. Consider a generic solution of the Newtonian N-body problem from *Newtonian initial data* in absolute space and time and project it down to $\mathcal{S}$ [3]. The projected curve will begin with some direction at some point in $\mathcal{S}$, i.e., with certain Machian initial data, and can be compared with the Machian evolution from the same data. There is a five-parameter family of Newtonian solutions in $\mathcal{S}$ that all emanate from the same data.

The five parameters correspond to information that is defined in absolute space and time but not in $\mathcal{S}$. Consider first motion in absolute space at right angles to the line joining two particles; it does not change their instantaneous separation $r_{ij}, \mathrm{d}r_{ij}/\mathrm{d}t = 0$, but gives them a centre-of-mass angular momentum $\mathbf{J}$. This $\mathbf{J}$, determined by two angles and one magnitude, is 'invisible' in the Machian initial data but has a strong effect on the subsequent evolution. A fourth datum is the system's total centre-of-mass energy E_{cm}. The fifth is the amount of kinetic energy associated with change of size of the system, measured by $\mathrm{d}I/\mathrm{d}t$, where

$$I = \sum_{i<j} m_i m_j r_{ij}^2 / M \tag{1}$$

is the centre-of-mass moment of inertia (M is the total mass). These data too are 'invisible' in Machian initial data but affect the evolution in $\mathcal{S}$. Having shown that Machian theories will be superior if they exist, I must now present the universal method for constructing them.

3 Best-Matched Machian Theories

3.1 The Elimination of Time

Jacobi [8] gave the first mathematically correct formulation of the principle of least action as a *geodesic principle* on the Newtonian configuration space Q:

$$\delta A_{\mathrm{J}} = 0,\ A_{\mathrm{J}} = 2\int \sqrt{(E-V)\sum_i \frac{m_i}{2}\delta\mathbf{x}_i\cdot\delta\mathbf{x}_i}. \tag{2}$$

Here E is the total energy of the system, V is its potential energy, and $\delta\mathbf{x}_i$ is the increment of the Cartesian coordinate of particle i in absolute space, which Jacobi accepted as given. Time is absent from (2), which determines the orbits of the system in Q with energy E. Jacobi used energy conservation to determine the speed in orbit through the relation

$$T = E - V,\ \text{where}\ T = \frac{1}{2}\sum_i m_i \frac{\delta\mathbf{x}_i}{\delta t}\cdot\frac{\delta\mathbf{x}_i}{\delta t}. \tag{3}$$

But for a closed system without an external time, (3) can be inverted to define an emergent time in terms of the $\delta\mathbf{x}_i$. We rewrite Jacobi's action in the parametrized form

$$\delta A_{\mathrm{J}} = 0,\ A_{\mathrm{J}} = \int \mathcal{L}\,\mathrm{d}\lambda,\ \mathcal{L} = 2\sqrt{(E-V)T_\lambda},\ T_\lambda = \sum_i \frac{m_i}{2}\mathbf{x}_i'\cdot\mathbf{x}_i',\ \mathbf{x}_i' = \frac{\mathrm{d}\mathbf{x}_i}{\mathrm{d}\lambda}. \tag{4}$$

Here λ parametrizes the curve in Q; the action in (4) is manifestly reparametrization invariant. Moreover, E appears simply as a constant part of the potential, much like the cosmological constant in general relativity. The canonical momenta corresponding to (4) are

$$\frac{\partial \mathcal{L}}{\partial \mathbf{x}'_i} \equiv \mathbf{p}^i = \sqrt{\frac{E-V}{T_\lambda}} m_i \frac{\mathrm{d}\mathbf{x}_i}{\mathrm{d}\lambda}, \tag{5}$$

and the equations of motion are

$$\frac{\mathrm{d}\mathbf{p}^i}{\mathrm{d}\lambda} = -\sqrt{\frac{E-V}{T_\lambda}} \frac{\partial V}{\partial \mathbf{x}_i}. \tag{6}$$

The radical on the right depends on λ, but it is obvious to simplify (6) by choosing λ in such a way that $(E-V)/T_\lambda$ is always a constant. Equation (6) then becomes Newton's second law with a distinguished time variable emergent from a geodesic principle. It is the astronomers' operationally defined *ephemeris, or Newtonian, time* [9, 10, 2]. Its increment δt

$$\delta t = \sqrt{\frac{\sum_i m_i \delta\mathbf{x}_i \cdot \delta\mathbf{x}_i}{2(E-V)}} \tag{7}$$

is the mass-weighted hypotenuse of the displacement of the system in Q divided by $\sqrt{2(E-V)}$. As Mach insisted, time is derived from change; it is redundant in a closed system. Its absence has consequences; the canonical momenta corresponding to (4) must satisfy the constraint

$$\sum_i \frac{\mathbf{p}^i \cdot \mathbf{p}^i}{2} = E - V. \tag{8}$$

The standard explanation for this quadratic constraint is the reparametrization invariance of (4). But this mistakes formalism for substance. What counts is the physics, the geodesic nature of the theory, the initial condition for which is an initial direction at a point in Q. The canonical momenta are essentially direction cosines multiplied by $\sqrt{2(E-V)}$; if direction cosines are squared, they sum to unity. The same effect produces the quadratic constraint (8).

We shall see that there is a very similar quadratic constraint in the Hamiltonian representation of general relativity. It leads to the 'problem of time', which people have tried to understand by analogy with parametrized particle dynamics. But that is a far less appropriate model than Jacobi-type dynamics, and another of my research projects will be to use Jacobi dynamics to study the problem of time.

3.2 The Elimination of Absolute Space and Scale

Jacobi's Principle defines a timeless theory on Q. To obtain a fully Machian theory, on $\mathcal{S}$, we need to define a 'distance' between any two nearly identical shapes A and B in $\mathcal{S}$. There is a universal way to do this [2]. Take for example N particles in Euclidean space $\mathcal{E}$ and for the moment allow scale. Represent A and B in Cartesian frames by $\mathbf{x}_i$ and $\bar{\mathbf{x}}_i$. There are no markers in $\mathcal{E}$ to identify its points, so we cannot say that particle i has made the displacement $\delta\mathbf{x}_i = \bar{\mathbf{x}}_i - \mathbf{x}_i$ as Newton did when he made positions absolute. Denying his fiction, we can act on $\bar{\mathbf{x}}_i$ with the translation and rotation generators, obtaining different $\delta\mathbf{x}_i$. This is the relationist's dilemma.

To attack it, let us seek a quantitative objective difference d_{AB} of A and B. It can depend only on the separations r_{ij} and $\bar{r}_{ij}$, and we find it as follows. Keep the representation of A by the $\mathbf{x}_i$

fixed; use the Euclidean generators to shift $\bar{\mathbf{x}}_i$ in all possible ways; for each of them calculate

$$d_{\rm AB}^{\rm trial} = \sum_i \sqrt{m_i(\bar{\mathbf{x}}_i - \mathbf{x}_i)\cdot(\bar{\mathbf{x}}_i - \mathbf{x}_i)} = \sum_i \sqrt{m_i \delta\mathbf{x}_i \cdot \delta\mathbf{x}_i}; \tag{9}$$

and find its unique minimum, which exists because (9) is positive definite.

For obvious reasons the minimum can be called the *best-matched* distance $ds_{\rm AB}^{\rm bm}$ in $\mathcal{S}$ between A and B. Such a best-matched metric is the minimal background-independent generalization needed to pass from geometry, founded on the congruence of unchanging figures, to dynamics, in which the figures, ultimately shapes of the universe, must change; the minimum $ds_{\rm bm}$ of (9) measures the *incongruence* of the two configurations. I believe that the principle of least action is an unrecognized *principle of least incongruence* based on best matching shapes of the universe.

Best matching is universal because it can be applied whatever geometrical binding principle holds a shape together and defines the shape space. Such a binding principle always leads to a redundant representation in a manifold on which one and the same shape is represented by points on a group orbit. Just as with particle configurations, there is no canonical connection between the orbits of different shapes. Each orbit is a fibre above a shape in the base space $\mathcal{S}$, and *a priori* there is no distinguished section connecting points in different fibres. Best matching acts with the group generators on the fibres to shift one relative to its neighbour into the best-matched position. A background-independent best-matched section is obtained using nothing but the shapes.

The measure of incongruence (9) is not unique. To see this, suppose we attempt to best match configurations when there is no scale; then we have to allow changes of relative scale between A and B. But (9) has the dimensions [length]^2 and can be made arbitrarily small by scaling. To prevent this, we must minimize a dimensionless expression. One candidate is

$$\sqrt{\sum_i \frac{m_i \delta\mathbf{x}_i \cdot \delta\mathbf{x}_i}{I^{1/2}}}, \tag{10}$$

where I is the moment of inertia (1), but other choices could be made. I shall give the name *incongruence measure* to any definite choice. Before considering possibilities, let us see how Newton's absolute space and time will effectively emerge from best matching.

Let a curve in S begin at shape A represented by initial coordinates $\mathbf{x}_i^{\rm in}$. Choose an incongruence measure and work along along the curve from A, best matching each shape with respect to its predecessor. This 'stacks' each shape in its best-matched position relative to A and establishes a best-matched *Machian equilocality relation* through the stack; it will say which point in each successive configuration is 'at the same position' as a given point in A.

Newton introduced absolute space to be able to say that force-free particles move rectilinearly. This is the essence of his law of inertia. One needs to say when two points are at the same position in space at different times; one needs an equilocality relation. Newton did not believe that the constantly changing relative positions of the bodies in the universe could provide it. Best matching shows that he was wrong in the case of a finite universe.

To see how a 'separation in time' can arise, we need to define actions. In the case with scale we only need to change the Jacobi action (2) into the Machian action

$$A_{\rm M} = 2\int_{\rm A}^{\rm B} \sqrt{(E-V)\sum_i \frac{m_i}{2}\delta\mathbf{x}_i^{\rm bm}\cdot\delta\mathbf{x}_i^{\rm bm}} \tag{11}$$

along trial curves between fixed configurations A and B in $\mathcal{S}$; $\delta\mathbf{x}_i^{\rm bm}$ are the best-matched displacements. The action (11) is background independent; no absolute equilocality or time is presupposed.

The Machian equilocality relation is determined by the configurations alone. The Euclidean representation space is not a background; it expresses the unity of the universe. The potential V must clearly depend only on the instantaneous configuration, i.e., on the r_{ij}. If we deny scale, the action (11) must be restricted further; it will only be scale invariant if $E = 0$ and V is homogeneous of degree -2. The separation in time associated with (11) is of course its ephemeris time.

Let me end by noting how readily both absolute time and space can be eliminated. We shall see soon that in general relativity they are eliminated in precisely this manner, which is therefore seen to be the irreducible basis of Einstein's theory. I believe it is essential to base any attempt at quantization of general relativity on explicit recognition of this fact. My research proposals are all made with this in mind.

4 The Machian Constraints

The best-matched displacements in (11) are obtained by varying with respect to the symmetry group generators [3, 11]; this leads to constraints on the canonical momenta $\mathbf{p}^i$ of the $\mathbf{x}_i$:

$$\mathbf{P} \equiv \sum_i \mathbf{p}^i = 0 \text{ (due to translations)}, \tag{12}$$

$$\mathbf{J} \equiv \sum_i \mathbf{x}_i \times \mathbf{p}^i = 0 \text{ (rotations)}, \tag{13}$$

$$D \equiv \sum_i \mathbf{x}_i \cdot \mathbf{p}^i = 0 \text{ (dilatations)}. \tag{14}$$

These best-matching constraints are all *linear* in the $\mathbf{p}^i$. The *quadratic* constraint (8) has its unrelated geodesic origin. For (12) and (13) to propagate, V can depend only on r_{ij}; for (14) to propagate, $E - V$ must be homogeneous of degree -2, which enforces $E = 0$.

It is natural to represent the Euler–Lagrange equations, derived in an arbitrary λ-dependent Cartesian frame, in the *distinguished representation* with best-matched stacking of the successive configurations and λ as the ephemeris time. The equations of motion are then Newton's! But they are constrained to be *very* special: by (13) to have vanishing centre-of-mass angular momentum and by (14) to have vanishing $D = \dot{I}/2 \Rightarrow I = \text{constant}$. A conserved moment of inertia I is virtually never considered in the literature; it requires $E = 0$ and V to be homogeneous of degree-2.[2]

Thus best-matching implementation of Mach's Principle leaves Newton's laws intact but strongly restricts their allowed solutions through (13) and (14) [by Galilean relativity (12) restricts V but not the solution set]. In the case with scale, the constraint (8) is also restrictive, allowing only Newtonian solutions with the fixed energy E. However, scale invariance already enforces $E = 0$, as I noted, and in this case (8) merely eliminates the fiction of absolute time.

The quadratic momentum constraint (8) does however always play one important role: it reflects fully and explicitly the particular incongruence measure that has been used in best matching. This is entirely hidden in the linear momentum constraints (12)–(14). But they in their turn reveal the kinds of best matching that led to their appearance; there is a one-to-one correspondence between the group generators used to best match and the form of the constraints. Taken together, the

[2] It is shown in [3] that one can recover all the standard Newtonian forces exactly by dividing their respective potentials by appropriate powers of I in order to make them have dimensions $[\text{length}]^{-2}$. One recovers Newtonian theory with additional forces that are very weak in realistic models of an island universe and ensure that its moment of inertia remains constant.

constraints say everything about the theory. Let me emphasize once more that the quadratic and linear constraints play different roles and have different origins.

The particle constraints have very close analogues in general relativity; this makes particle models excellent toy models for quantum gravity. They will be an important part of my research project.

5 The Machian Constraints in Geometrodynamics

The definition of Mach's Principle is universal and is particularized by the choice of either a relative space $\mathcal{R}$ with scale or a scale-free shape space $\mathcal{S}$. The basic structure of best matching is also universal and is particularized by the choice of the incongruence measure. In this section, I shall review very briefly the results obtained for superspace [2, 11], conformal superspace (CS) [12], and CS+V [6]. The extensions to include matter will also be mentioned.

In the particle dynamics, the Euclidean translation, rotation, and dilatation groups, which are finite-dimensional Lie groups, sufficed to implement best matching. But a Riemannian 3-geometry is characterized by infinitely many degrees of freedom – three per space point – and it is therefore necessary to best match with respect to the infinite-dimensional group of 3-diffeomorphisms. The universality of best matching is reflected in the appearance in the best-matched expressions of the Lie derivatives of the 3-metric g_{ij} and whatever matter fields occur as well. All fields have uniquely defined Lie derivatives; this leads to the power and precision of best matching. I shall again give the best-matched form of the action; the manner in which the group generators appear in it is given in [11, 12, 6]. We shall see that in the more intricate setting of infinite-dimensional Lie groups the choice of the incongruence measure is strongly restricted; consistent best matching becomes a powerful tool for finding theories.

5.1 Best Matching on Superspace and Machian Geometrodynamics

Consider two slightly different Riemannian 3-metrics $g_{ij}(x)$ and $\bar{g}_{ij}(x) = g_{ij}(x) + \delta g_{ij}(x)$ on S^3. Newton's introduction of absolute space is like saying that x is the 'same point in space' for the two metrics; x defines equilocality. To avoid this fiction, we consider all equilocality relations generated by 3-diffeomorphisms of $\bar{g}_{ij}$. For each resulting δg_{ij}, we must extremalize an incongruence measure including the square root of a quadratic expression; it could have the form

$$\sqrt{\int \mathrm{d}^3x\sqrt{g}P \int \mathrm{d}^3x\sqrt{g}G^{ijkl}\delta g_{ij}^{\mathrm{bm}}\delta g_{kl}^{\mathrm{bm}}}, \tag{15}$$

with square root taken after integration over space (global square root), or

$$\int \mathrm{d}^3x\sqrt{g}\sqrt{PG^{ijkl}\delta g_{ij}^{\mathrm{bm}}\delta g_{kl}^{\mathrm{bm}}}, \tag{16}$$

with square root taken first (local square root). In both, the potential-like term P is a 3-scalar function of g_{ij} and its spatial derivatives, and G^{ijkl} defines a supermetric on Riem. In the simplest case it is ultralocal in g^{ij} and generates two independent terms: $g^{ik}g^{jl}\delta g_{ij}^{\mathrm{bm}}\delta g_{kl}^{\mathrm{bm}}$, the 'trace of the square $\delta g_{ij}^{\mathrm{bm}}$', and $g^{ij}g^{kl}\delta g_{ij}^{\mathrm{bm}}\delta g_{kl}^{\mathrm{bm}}$, the 'square of the trace'.

Because g_{ij} has independent degrees of freedom at each x, there will be canonically paired g_{ij} and p^{ij} at each x. Moreover, arbitrary 3-vector fields generate the 3-diffeomorphisms. Instead of global constraints like (12)–(14), we shall, whatever incongruence measure is chosen, have best-matching constraints with the universal form dictated by the diffeomorphism generators:

$$p^{ij}{}_{;j} = 0, \tag{17}$$

where the semicolon denotes the covariant derivative with respect to g_{ij}. This infinite set of linear momentum constraints, one at each space point, generalizes the particle constraints (12) and (13) (but not (14) because g_{ij} has scale). Now we must choose between (15) and (16).

The 'global' choice (15) corresponds straightforwardly to the Jacobi action (11) and leads to a single quadratic 'direction-cosine' constraint; the local (16) is a more sophisticated generalization and leads to a quadratic constraint at each x of the form

$$p_{ij}p^{ij} + ap^2 = gP, \; p = g_{ij}p^{ij}, \; a = \text{constant}, \tag{18}$$

where the undetermined a reflects the presence of the two independent terms in G^{ijkl}.

Two related arguments can be given for (16): it eliminates action at a distance, and the linear (17) is then matched by the quadratic (18) at each x. The local choice is also very interesting because Euler–Lagrange propagation of (18) is by no means guaranteed and becomes a powerful theory-selection principle [11]. The linear (17) always propagates because (16) already satisfies the strong requirement of 3-diffeomorphism invariance. However, (16) merely generalizes the timeless Jacobi action (2). Local elimination of time is the origin of (18); no symmetry has been built in to ensure that it propagates. In fact, it only does so if in (18)

$$a = -1/2 \text{ and } P = \Lambda + bR, \tag{19}$$

where Λ is a constant, R is the 3-scalar curvature, and $b = 1, 0, \text{or} -1$. The theory with $b = 1$ is general relativity with cosmological constant Λ, and the action corresponding (16) in this case is the Jacobi-type Baierlein–Sharp–Wheeler action [13] of general relativity; $b = 0$ corresponds to so-called strong gravity, and $b = -1$ to Euclidean general relativity. We have a remarkably direct Machian derivation of general relativity for a closed universe. But now, unlike the global constraints (12)–(14), the constraints (17) and (18) hold locally and are in fact the $G^{0i} = 0$ and $G^{00} = 0$ Einstein field equations respectively in Hamiltonian form. Moreover, the constraints encapsulate the entire theory, so that confirmation of Einstein's theory anywhere suggests that its origin is profoundly Machian in the sense defined here. We can probably never know for sure whether the universe is spatially closed, but the success of best matching, which presupposes it, is support for the conjecture. This is part of the motivation for my research project.

If one attempts to couple matter fields to Machian geometrodynamics, the constraints are modified by the addition of matter terms [11]. It is easy to ensure that the modified constraint (17) propagates, but propagation of the modified (18) imposes strong conditions: 1) the matter fields must have the same light cone as the geometrodynamics – the full causal structure of general relativity is imposed; 2) for a vector field, no less than four conditions are imposed and together force it to be the Maxwell field coupled to gravity; 3) a set of vector fields can interact simultaneously with one another and with the Machian geometrodynamics only if they are Yang–Mills gauge fields [14]. These results are obtained subject to certain simplicity assumptions [15], but it is remarkable that the Machian principles lead so readily to general relativity, its full causal structure (universal light cone), and gauge theory as a single intimately interconnected package – and all through an obvious way to avoid Newtonian-type absolute structures. I can now say why I advocate the Machian approach to quantum gravity in preference to others:

• String theory is not at present background independent and therefore does not focus on the key property of general relativity. If it is ever found, M theory must surely be background independent. There is therefore a case for looking at theories that already are.

• Loop quantum gravity (LQG) is based on spin networks embedded in a bare manifold. One cannot best match such entities. They do not maintain the integrity of 3-geometry and therefore cannot be varied infinitesimally in the manner require for best matching.

• The causal-set approach is fundamentally discrete and like LQG attempts to do without the integrity of 3-geometry. It identifies the causal structure as the most important property of Einsteinian spacetime. But we have seen that the causal structure emerges from best matching, which solves the most basic problem of all: how is motion to be defined?

5.2 Scale-Invariant Machian Geometrodynamics

The quadratic constraint (17) propagates under the conditions (18) because the BSW action [13] has a remarkable invariance in addition to the 3-diffeomorphism invariance consciously put into (17). It is the foliation invariance of general relativity and emerges in the Machian approach through the local choice of the square root needed to eliminate an external time. Now a 3-geometry has three physical degrees of freedom per space point, while the foliation invariance is universally agreed to reduce their number to two. However, unambiguous identification of them has proved to be difficult and controversial. The Machian perspective suggests rather naturally that the true geometrodynamic variables are the two conformal degrees of freedom per space point in a 3-metric g_{ij}. In this case, one would have a fully scale invariant and 3-diffeomorphism invariant theory.

This is the idea behind [12], which was to seek a geometrodynamic theory that, like general relativity, is 3-diffeomorphism invariant but posits 3-conformal invariance instead of foliation invariant. A minor modification of the BSW action leads to a geodesic theory in conformal superspace CS. Besides the necessary linear 3-diffeomorphism constraint ${p^{ij}}_{;j} = 0$ and a quadratic constraint very similar to that of general relativity, the theory has an additional linear momentum constraint $p = g_{ij}p^{ij} = 0$ analogous to the scaling constraint (14). This forces the volume of the universe to remain constant, and it is hard to imagine the theory rivalling the big-bang explanation of cosmological observations. It also has an unappealing weak form of action at a distance.

My collaborators and I also explored [6] the possibility of marginally weakening the requirement of 3-conformal invariance to volume-preserving 3-conformal invariance: the local scale factor of the metric g_{ij} becomes gauge, but the total volume of the universe is physical and can change; this yields a theory in CS+V. From the Machian perspective, the volume-preserving condition is bizarre – the theory would be *perfectly* Machian without it – but the resulting theory turns out to be general relativity in the distinguished constant-mean-curvature (CMC) foliation that York [16] used to obtain initial data for general relativity that satisfy the initial-value constraints. The Machian best matching in such a theory leads to not only the two familiar constraints of general relativity but also to York's condition $g_{ij}p^{ij} = p =$ constant that enforces the CMC foliation and the lapse-fixing condition that ensures propagation of it. They all come together as one best-matching package.

York's conditions have hitherto been regarded as gauge-fixing conditions convenient for finding consistent initial data; it is thought provoking that a physically inspired best-matching requirement so close to being perfectly Machian should lead not only to general relativity but simultaneously to the only known reliable method for solving its initial-value problem. It is also worth noting that specification of a direction at a point in CS plus one single number – the York time (essentially the ratio of the energy in expansion of the universe to the energy in all the changes in its local shapes) – leads to complete determination of the local inertial frames of reference and local proper lengths and times (the latter as local analogues of ephemeris time). They are not in the true ontology in CS+V but emerge from best matching as artefacts of the distinguished representation.

Both the conformal theories – in CS and CS+V – put a question mark over the relativity principle and foliation invariance. They define universal simultaneity uniquely by the dynamical procedure needed to define change – best matching – and yet retain the universal light cone that is the distinguishing feature of special relativity. It has always been recognized that in special relativity one could claim that there is a unique distinguished frame; the difficulty is that no dynamical definition of it exists, so that two independent observers could never agree which it is. It is also often pointed out that the microwave background provides a kinematic state of rest, though not one that can be unambiguously and exactly defined. In contrast the CMC foliation is exact and dynamically dictated in the Machian approach. Moreover any successful identification of the long sought for two true degrees of freedom per space point in general relativity must break its foliation invariance. The dynamically defined CMC rest frame even has the potential to eliminate residual unease about the twin paradox, making it unnecessary to attribute it, consistently but mysteriously, to the structure of invisible Minkowski space. Clocks would slow down and rods contract relative to the CMC foliation. The principle of least incongruence acting on shapes of the universe would explain all such phenomena.

But the need to specify the York time – *one* number not contained in a point and a direction in CS – remains, hence the conditional words 'might, could, would'. The proximity to complete understanding is tantalizing.

6 Proposed Research and Methodology

Advancing the Machian programme requires conceptual clarification and identification of specific issues prior to the technical work that is also planned. In the past, this has been the aspect of research in which I have played the most useful role. For this reason, the bulk of the funding for which I am applying is to pay for mini-workshops and one-to-one interactions that have as their aim identification of such technical issues on which my more technically proficient collaborators can then work. This is how the joint papers [2, 11, 12, 6], which I regard as among my best work, came into being. With more collaborators expressing an interest in the Machian approach, I am keen to continue and expand this kind of activity. In addition, I find that all such work is helping greatly in the writing of my book [22], which I hope will be a definitive account of the Machian programme. It has many close and illuminating connections with gauge theory as well as general relativity.

In my mind it is important that within the Machian approach outlined above long-standing issues can be addressed with nontrivial particle toy models in which standard quantum mechanics can be used. This is because the solutions of the classical Machian particle models are very special solutions of Newtonian dynamics, so that the quantum theory corresponding to them should in some sense be contained within already existing quantum theory. In the following subsections, I shall spell out some of the research directions that appear to me most promising and on which I propose to use a successful grant.

6.1 Jacobi Dynamics as Guide to Solution of the Problem of Time

General relativity was early seen [20] to be an 'already parametrized' theory with a concomitant 'problem of time'. In my view it is most unfortunate that parametrized particle dynamics, with a parabolic as opposed to purely quadratic Hamiltonian constraint, was believed to indicate the way to its solution. It suggested that one should 'deparametrize' general relativity and find an 'internal time' among its variables. Except for the York time, which is certainly interesting but leads to nonconservative dynamics, nearly half a century of attempts has produced nothing of proven value. From my presentation of the Machian approach, it is evident that Jacobi dynamics is the correct toy model for studying the problem of time in general relativity.

Having been alerted to Jacobi dynamics through my papers, Hans Westman and Sean Gryb of the Perimeter Institute began to study it through path-integral methods a few months ago. This is a typical example of the way in which I have been able to make a useful contribution. The three of us have now started to collaborate. Hans Westman has already spent 10 days with me, and Sean will visit shortly if my FQXi mini-grant is successful. This has every indication of developing into a fruitful collaboration, one of the main aims of which is to discover how laboratory quantum mechanics could emerge from a Wheeler–DeWitt equation. In particular, how does the quantum-mechanical 'absolute' time arise? Classical Jacobi dynamics indicates clearly that it must appear as ephemeris time, which is measured by *all* changes in the universe, not by some artificially identified internal time variable. The path-integral techniques being developed by Sean and Hans have real promise and should be the first real step to the definitive resolution of the 'problem of time': there will be no time in the foundations of the theory, and the specific way in which it emerges as ephemeris time will be identified.

6.2 The Role of the Linear Momentum Constraint

Few quantum-gravity studies have included the linear momentum constraint (LMC); it is difficult to handle and is mistakenly regarded as less fundamental than the quadratic momentum constraint (QMC). There have been umpteen studies of mini-superspace models without the LMC, but they cannot capture the essential dynamics. The particle models could be far more illuminating; they have non-trivial LMCs, are calculable for the 3-body case, and may be amenable to qualitative methods in the N-body case. They are very useful models for studying the emergence of inhomogeneities in the early universe on a more fundamental basis than inflation. My collaborator Edward Anderson has already made useful progress in this direction, in which little other work has been done. I plan mini-workshops with Edward and others in order to push forward this work energetically. Such work will also test the common assumption that a closed universe has a Hilbert space of states. But what if only a unique state exists, and a Hilbert space cannot be found by formal Dirac quantization? A study of simple particle models may indicate a need to completely reconsider the canonical approach to quantum gravity.

In this connection I am already collaborating with Henrique Gomes at Nottingham University and exploring the following idea. If there is a wave function Ψ of the universe, its determination surely involves the best-matching metric on some relative configuration space. Indeed, *two* extremalization processes may determine Ψ: best matching and extremal selection among wave functions defined wrt that metric. We aim to test this possibility on few-particle models.

6.3 Observables in Canonical Quantum Gravity

Most authors – Kuchař [17] is a notable exception – affirm that observables in canonical quantum gravity must commute with not only the LMC, which is clear, but also with the QMC. They cite Dirac's 'theorem': all first-class primary constraints generate gauge transformations that do not change the physical state [18]. But Dirac proved this (correctly) for systems with a true Hamiltonian to which an LMC-type constraint is adjoined. He noted later that in fully constrained (parametrized) systems at least one constraint must generate motion but seems not to have recognized this as a counterexample to the 'general proof'. The fact is that such a constraint carries the system along its dynamical curve to physically different configurations, as is obvious with the QMC in Jacobi mechanics. The only gauge aspect is in fictitious 'changes of speed' with which this happens. Real speed requires absolute time; the quadratic constraint demonstrates that no such time exists in closed-universe general relativity. This confusion has largely arisen because Dirac [18] created a general formalism applicable to all constraints; this has made his work immensely valuable, but has created the false impression that all first-class constraints are much the same in nature. This is surely uncritical and simply wrong; it is necessary to consider why specific constraints are there and what they do. It then becomes clear that the LMC and QMC have different origins and play different roles. Observables need only commute with the LMC. One of my research projects is to identify such observables in the particle models as reliable guides to the kind of observables to be expected in quantum gravity. It should be noted that best matching compares complete configurations of the universe and suggests that any observable must be a function on them. One of the aims of my project is to prove that no local observables can exist in quantum gravity; if confirmed, this would show that current approaches, which aim to construct local observables, are misguided.

6.4 The Cosmological Interpretation of Quantum Mechanics

The Machian perspective with its clearly defined configuration-space ontology is ideally suited for interpreting quantum mechanics in the closed-universe context. For example, the demonstration that the entire dynamics of general relativity, including the only viable method of solving its initial-value, arises from best matching suggests that the configuration space has primacy over the momentum space. It should be noted that the Dirac–Jordan quantum transformation theory relies on the momentum and configuration spaces having the same number of dimensions. But this arises from the absolute time in Newtonian dynamics, without which momenta become directions without magnitude. In the context of many-worlds interpretations, primacy of the configuration space should settle the preferred-basis issue; the configuration basis is preferred, as Bell always insisted on epistemological grounds. The Machian approach complements epistemology with a theoretical argument. Further, the Machian approach has the same ontological basis as the de Broglie–Bohm theory, and I plan mini-workshops on the cosmological interpretation of quantum mechanics, to which I shall invite Simon Saunders, Antony Valentini, Rob Spekkens, Hans Westman, and Terry Rudolf. (The first three are FQXi members and grant awardees, and Hans Westman is short listed for the current grant ground; all but the second have already taken part in mini-workshops at my home.) One key issue to be attacked with them is this: if one does take some form of many-worlds or de Broglie–Bohm interpretation seriously, what properties must the wave function of the universe have in order for us to be able to recover the outcomes of repeated measurement experiments in the laboratory? This question is closely related to the problem of the arrow of time. I expect work on it to give substance to my conjecture [21] that the arrow of time arises ultimately from the unavoidable and strongly pronounced asymmetry of any relative configuration space likely to be suitable to describe the universe.

6.5 Two Possibilities for Quick Breakthroughs

The creation of quantum gravity is likely to be a long haul, but I see two 'paradoxes' that could lead to early and perhaps dramatic breakthroughs.

First, quantum mechanics uses complex numbers, but the Wheeler–DeWitt equation is real. Whence come the complex numbers? I pointed out this conundrum, which has escaped most theoreticians, in [19]. It is a puzzle whose resolution could be as dramatic as Dirac's finding of spinors when unifying quantum mechanics and special relativity. A thorough study of the role of complex numbers is warranted independently. What empirical evidence enforces their use? Not enough physicists are concerned about this. I plan research and mini-workshops on this topic.

Second, the search for scale-invariant theories of the universe has had valuable bye products, the most famous being Weyl's eventual discovery of gauge theory, but, as Weyl found, scale invariance seems to be blighted by a 'no-go curse'. In the Machian approach, why does general relativity fail *by just one number* – the York time – to be fully scale invariant? Perhaps we need to develop a new concept of space and distance that takes its inspiration from Descartes's conjecture that space and matter are one and the same thing: *res extensa*. This could eliminate the dichotomy between geometry and matter that so distressed Einstein ("fine marble as against rough timber") and lead at last to a truly unified theory of all the interactions. We may not need to change from particles to strings but instead to seek a different geometry. I intend to include study of the foundations of geometry and discussion workshops in my research programme.

6.6 Concluding Remarks

The Hamiltonian formulation of general relativity and Dirac's theory of constrained systems were both created around 50 years ago and provided the basis for the development of the initial Wheeler–DeWitt metric form of canonical quantum gravity [23]. Failure to make significant progress with it led eventually to Ashtekar's new variables and LQG; string theory developed simultaneously and independently. I have argued that both fail to take into account adequately the true dynamical basis of general relativity. I think that there is a strong case for reconsidering the metric approach, which is more conservative than both: it does not postulate string theory's hitherto unobserved dimensions with unpredictable properties, and it maintains the continuum as an essential feature. All the essential dynamics revealed by the Machian approach is contained at least implicitly in metric canonical gravity even if its true basis has not hitherto been properly recognized. The Machian approach sharpens the conceptual issues and provides nontrivial toy models on which technical progress can be made. This is the basis for my grant application.

In further support of it, I may add that my home – I am the lucky owner of the beautiful College Farm built in a village north of Oxford in 1659 – has proved to be an ideal venue for mini-workshops with a high success rate in terms of technical papers that have resulted from them. Several FQXi members have already participated in them, and I hope more will in the years to come. College Farm has also been a venue in which young researchers have interacted fruitfully with senior researchers. This activity will be significantly enhanced by the grant for which I am applying.

References

[1] Mach E *Die Mechanik in ihrer Entwicklung historisch-kritsch dargestellt*, Barth, Leipzig (1883); English translation: Mach E *The Science of Mechanics*, Open Court, Chicago (1960) (translation of 1912 German edition).

[2] Barbour J and Bertotti B "Mach's principle and the structure of dynamical theories" *Proc. R. Soc. A* **382** 295 (1982) (downloadable as PDF file from www.platonia.com).

[3] Barbour J "Scale-invariant gravity: particle dynamics" *Classical and Quantum Gravity* **20** 1543–1570 (2002) (arXiv:gr-qc/0211021).

[4] Fischer A “The theory of superspace” in: *Relativity* eds. M Carmeli, S I Fickler, and L Witten, Plenum, New York (1970).

[5] DeWitt B 1970 “Spacetime as a sheaf of geodesics in superspace” in: *Relativity* eds. M Carmeli, S I Fickler, and L Witten, Plenum, New York.

[6] Anderson E, Barbour J, Foster B Z, Kelleher B, and Ó Murchadha N “The physical gravitational degrees of freedom” *Classical and Quantum Gravity* **22** 1795 (2005) (arXiv:gr-qc/0407104).

[7] Poincaré H, *La Science et l'Hypothèse* (1902); English translation: *Science and Hypothesis*, The Walter Scott Publishing Co, London (1905)

[8] Lanczos C *The Variational Principles of Mechanics*, University of Toronto Press (1949).

[9] Poincaré H “La mesure du temps” *Rev. Métaphys. Morale* **6** 1 (1898); English translation: “The measure of time,” in: Poincaré H *The Value of Science*.

[10] Clemence G “Astronomical time” *Rev. Mod. Phys.* **29** 2 (1957).

[11] Barbour J, Foster B Z and Ó Murchadha N 2002 “Relativity without relativity” *Class. Quantum Grav.* **19** 3217; (2002) (arXiv:gr-qc/0012089).

[12] Anderson E, Barbour J, Foster B Z and Ó Murchadha N 2002 “Scale-Invariant Gravity: Geometrodynamics” *Classical and Quantum Gravity* **20** 1571 (2003) (arXiv:gr-qc/0211022)

[13] Baierlein R F, Sharp D H, and Wheeler J A “Three-dimensional geometry as a carrier of information about time” *Phys. Rev.* **126** 1864 (1962).

[14] Anderson E and Barbour J “Interacting vector fields in relativity without relativity” *Class. Quantum Grav.* **19** 3249 (2002) (arXiv:gr-qc/0201092)

[15] Anderson E “On the recovery of geometrodynamics from two different sets of first principles” *Studies in History and Philosophy of Modern Physics* **38**, 15 (2007).

[16] York J “Gravitational degrees of freedom and the initial-value problem” *Phys. Rev. Letters* **26** 1656 (1971); “Role of conformal three-geometry in the dynamics of gravitation,” *Phys. Rev. Letters* **28** 1082 (1972).

[17] Kuchař K “Canonical quantum gravity” in: *Proceedings of the 13th International Conference on Geneeral Relativity and Gravitation*, eds R J Gleiser, C N Kosameh, and O M Moreshi, IOP Publishers, Bristol (1992).

[18] Dirac P *Lectures on Quantum Mechanics*, Jeshiva University, New York (1964).

[19] Barbour J 1993 “Time and complex numbers in canonical quantum gravity” *Physical Review D* **47** 5422.

[20] Arnowitt R, Deser S, and Misner C W 1962 “The dynamics of general relativity” in *Gravitation: An Introduction to Current Research*, ed. L Witten, Wiley, New York, p. 227

[21] Barbour J *The End of Time*, Weidenfeld and Nicolson, London (1999) (OUP, New York (2000)).

[22] Barbour J *Mach's Principle, General Relativity, and Gauge Theory*, Oxford University Press, New York (forthcoming).

[23] DeWitt B S “Quantum theory of gravity. I. The canonical theory” *Physical Review* **160** 1113–1148 (1967).